# Understanding pathological semidefinite programs: how elementary row operations help

Semidefinite programs (SDPs) -- optimization problems with linear constraints and semidefinite matrix variables -- are some of the most useful and versatile optimization problems of the last three decades. They appear in combinatorial optimization, engineering, machine learning, to name just a few areas.

While SDPs are useful, they are often pathological: for example, they may not attain their optimal value, their optimal value may differ from that of their dual. Pathological SDPs are often difficult, or impossible to solve.

I will show that many of these pathologies can be understood using a surprisingly simple tool: we can transform SDPs to a standard form that makes the pathology trivial to recognize. The transformation mostly uses elementary row operations coming from Gaussian elimination. The standard forms have computational uses, for example, in several cases they help us recognize infeasibility.

The talk will rely only on knowledge of elementary linear algebra, and some basic convex analysis, which I will introduce.

Some of this work is joint with Minghui Liu. Some other parts are joint work with Yuzixuan Zhu and Quoc Tran-Dinh.